THE STRIP SIMULATION METHOD FOR COMPUTING ELECTRIC FIELD ON CONDUCTOR SURFACES

Yixin Yang, Daniel Dallaire J. Ma and F. P. Dawalibi
Safe Engineering Services & technologies ltd.
1544 Viel, Montreal, Quebec, Canada, H3M 1G4
Email:info@sestech.com Web site: http://www.sestech.com

Abstract

This paper proposes a new method, the Strip Simulation Method, for computing the electric field on the surfaces of conductors for power transmission systems. This method can also compute the electric field at any observation point in space. The effects of earth surface also have been taken into account. The computation results have been compared with those obtained using the well known successive image theory. The comparison of the results obtained for bundles of four conductors show that when the ratio of the conductor radius to the distance between conductors is greater than 1, the two methods give similar results. When it is smaller than 1, the Successive Image Method tends to overestimate the minimum gradient on the conductor for bundles with more than 2 conductors and the proposed method gives accurate results. A practical case of a transmission line has been studied in this paper. The electric fields on the conductor surface and at the earth surface have been computed. In this case, the proposed method and the image method give consistent results.

Keywords

Strip, Simulation, Charge, Potential, Electric Field, Conductor Surface, Bundle, Image

I. Introduction

For a long parallel transmission line system, the distribution of the electric field on the surface of conductors and in space can be treated as a two-dimensional distribution. Therefore it is possible to use two-dimensional methods to compute the electric field. The Strip Simulation Method assumes that the charge in the conductor gathers on the conductor surface. Each conductor surface is equally subdivided into several straight and flat strips. The cross section of the conductor forms a polygon, which approaches a circle as the number of strips increases. The points midway across each strip are chosen to be the matching points. The energization voltages are applied to these points to compute the charges of the strips.

There are several methods which can be used to evaluate the total charges of the strips. In this paper, the Markt and Mengele [1] charge evaluation method was used to evaluate the total charge of each phase, and then accurately compute the distribution of charges among the strips according to the principle that all the strips in each phase are at the same potential. In each strip, the charge is then equally distributed. When computing the electric field on the conductor surface, the observation points are located on the circumference of the conductor instead of on the strips. Once the charge on each strip is known, the electric field contributed by the strip on the conductor surface or at any observation point above the ground can be computed by integrating the contribution of the strip and its image; the summation of the field generated by all the strips gives the field at the observation point.

A practical example of a three-phase transmission line with two shield wires is studied. In this example, each phase consists of a bundle of four conductors. The electric fields on the conductor surface and at ground level have been computed by both the Strip Simulation Method and the successive image method. The computation results show that the electric fields computed by both methods are in good agreement. In this real case, the bundle radius is much larger than the conductor radius.

II. Computation Procedure

The computation procedure consists of three steps.

Step 1. Subdivide the conductor surfaces into strips

In any conductor, the charge must distribute at its surface. In order to compute the charge distribution, the conductor surface is simulated by a number of infinitely long flat strips. On each strip, the charge is assumed to be equally distributed.

The strips are of equal width. Assuming the surface of the conductor is subdivided into N strips, the cross section of the conductor is an N-sided polygon. The width S of a strip is

$$S = 2R\sin\frac{\pi}{N} \tag{1}$$

where R is the radius of the conductor.

Fig. 1 shows a conductor which is subdivided into six strips. Obviously, the more strips into which the conductor is divided, the closer the simulated polygon and the real conductor surface will be.

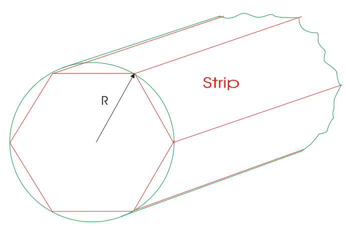

Fig. 1. The cross section of a conductor is simulated as a hexagon.

Step 2. Compute the charge on the strips

In principle, in order to obtain the charge on each strip, the potential coefficient matrix should be found first. This matrix contains elements of potential coefficients of all the strips in the system. The elements are determined by the positions of the strips and their images. The matrix equation is then solved to get the charges on each strip. This method may need to solve the full matrix, which is a large matrix, equation when there are many conductors and a large number of strips are simulated. This however can be avoided if the conductors are grouped in clusters (phases) that are remote from each other as is the case of phase bundles of most transmission lines. This is explained further in the next paragraph.

In this paper, the total charge Q_{ph} of each phase is first evaluated using the image method and equivalent radius method proposed by Markt and Mengele [1], then distributed to each strip. This method only needs to solve a smaller potential coefficient matrix for each phase instead of solving the full potential coefficient matrix for the whole system.

In order to distribute the charges to the strips, a relative potential coefficient matrix $[P]$ is computed for strips belonging to the phase. Let a vector $[q]$ represent the charges on the strips: $[q]$ is unknown and to be determined. $[V]$ is the known vector of energization voltage of the phase. Obviously, all of the elements in vector $[V]$ are the same as the voltage of the phase. The relationship between $[P]$, $[q]$ and $[V]$ is expressed in the following matrix equation

$$[P]\cdot[q] = [V] \tag{2}$$

Solving (2) gives the charges on the conductor strips. The total charge of the phase is the summation of the charges on all the strips belonging to the phase

$$Q_{tot} = \sum q_i \tag{3}$$

Usually, there is a small difference between Q_{ph} and Q_{tot}. The scaling factor F is defined as

$$F = \frac{Q_{ph}}{Q_{tot}} \tag{4}$$

The adjusted charge of the strip is q_i'

$$q'_i = Fq_i \tag{5}$$

The sum of q_i' is equal to Q_{ph}. The charge q_i' is used to compute the electric field. If necessary, the procedure can be iterated to adjust the charge.

Step 3. Determine the electric field at the surface of the conductors

Once the charges on all the strips are found, the electric field on the circumference of the conductors and any observation points can be computed by using the integration method. In Fig. 2, assuming that there is an observation point P located at x_0, y_0. For a line charge on the strip, its contribution to the field at P is

$$\vec{E} = \frac{\Delta q}{2\pi\varepsilon_0 |r|}\hat{a}_r \tag{6}$$

where r is a vector from the line charge to the point P, $\hat{a}_r$ is the unit vector of r, and Δq is the charge of the line with width of du

$$\Delta q = \delta du \tag{7}$$

in which $\delta = q / S$, the per unit charge, S is the width of the strip.

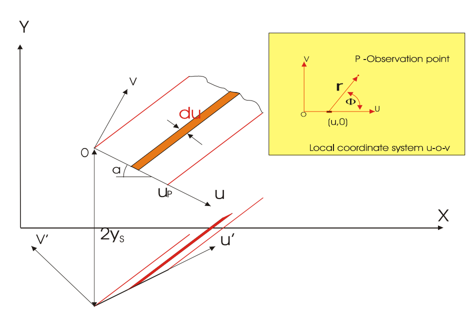

Fig. 2. Local coordinates for computing the electric field on the conductor surface.

In Fig. 2, in the local coordinate system u-o-v, the electric field is

$$\vec{E} = E_u \hat{a}_u + E_v \hat{a}_v \tag{8}$$

where Eu and Ev are components of electric field parallel to the u and v axes, respectively. $\hat{a}_u$ and $\hat{a}_v$ are unit vectors of axis u and axis v respectively. For any infinitely thin line on the strip, the modulus of the horizontal and vertical components of the field in the local coordinate system can be calculated as

$$|E_u| = \frac{\lambda \cos \Phi}{|r|}$$
$$|E_n| = \frac{\lambda \sin \Phi}{|r|} \tag{9}$$

where $\lambda = \dfrac{\Delta q}{2\pi\varepsilon_0}$. We transform the observation point x_0, y_0 in the x-y coordinate system to the local coordinate system u-o-v

$$u = (x - x_0)\cos\alpha + (y - y_0)\sin\alpha$$
$$v = -(x - x_0)\sin\alpha + (y - y_0)\cos\alpha \tag{10}$$

where α is the angle between the x-axis and the u-axis. In order to take the image of the strip into account, the observation point x_0 ,y_0 in the x-y coordinate system should also be transformed to the local coordinate system u'-o'-v'

$$u' = (u + 2y_s \sin\alpha)\cos 2\alpha - (v + 2y_s \cos\alpha)\sin 2\alpha$$
$$v' = (u + 2y_s \sin\alpha)\sin 2\alpha + (v + 2y_s \cos\alpha)\cos 2\alpha \tag{11}$$

Rewrite (11) as

$$u' = u\cos 2\alpha - v\sin 2\alpha - 2y_s \sin\alpha$$
$$v' = u\sin 2\alpha + v\cos 2\alpha + 2y_s \cos\alpha \tag{12}$$

The electric field generated by the strip is computed by integrating the contribution to P over the width of the strip

$$E_u = \lambda\left\{\int_0^{u_p} \frac{(u_m - u)du}{(u_m - u)^2 + v_m^2} - \int_0^{u_p} \frac{(u'_m - u')du'}{(u'_m - u')^2 + v'^2_m}\cos 2\alpha - \int_0^{u_p} \frac{v'_m du'}{(u_m - u)^2 + v_m^2}\sin 2\alpha\right\}$$
$$E_n = \lambda\left\{\int_0^{u_p} \frac{v_m du}{(u_m - u)^2 + v_m^2} - \int_0^{u_p} \frac{v'_m du'}{(u'_m - u')^2 + v'^2_m}\cos 2\alpha + \int_0^{u_p} \frac{(u'_m - u')du'}{(u'_m - u')^2 + v'^2_m}\sin 2\alpha\right\} \tag{13}$$

In (13), u_p is the width of the strip. The first term is the contribution of the strip, the second and third terms are contributions from the image. The electric field can be represented as

$$E_u = -\frac{1}{2}\lambda \ln\left[\frac{(u_m - u_p)^2 + v_m^2}{u_m^2 + v_m^2}\right] - \frac{1}{2}\lambda\cos 2\alpha \ln\left[\frac{(u'_m - u_p)^2 + v'^2_m}{u'^2_m + v'^2_m}\right]$$
$$-\lambda tg^{-1}\left[\frac{u'_m u_p}{u'^2_m + v'^2_m - u_p u'_m}\right]\sin 2\alpha$$
$$E_v = \lambda tg^{-1}\left[\frac{u_m u_p}{u_m^2 + v_m^2 - u'_m v_m}\right] - \lambda tg^{-1}\left[\frac{u'_m u_p}{u'^2_m + v'^2_m - u_p u'_m}\right]\cos 2\alpha$$
$$-\frac{1}{2}\lambda \sin 2\alpha \ln\left[\frac{(u'_m - u_p)^2 + v'^2_m}{u'^2_m + v'^2_m}\right] \tag{14}$$

The electric field at any observation point can be obtained by adding up the contributions from all the strips.

III. Validations and Comparisons

Let us consider a phase bundle with four conductors, as shown in Fig. 3. The center of the bundle is located 50 meters above ground. The radii of all conductors in the bundle are 1 cm. The bundle is energized with a voltage of 1 kV. In the figure, *D* represents the distance between the neighboring conductors and *R* is the radius of the conductor. The electric field on the surface of Conductor 1 is computed with various ratios of D/R. The number of the strips simulated for each conductor is 512, a large number to ensure the maximum accuracy. The observation profile is placed on the surface of Conductor 1, starting from the 3 o'clock position on the circumference, running in a counterclockwise direction.

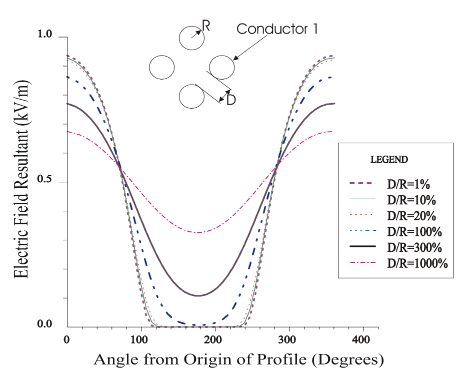

Fig. 3. Electric field on the surface of Conductor 1, based on the Strip Simulation Method; the legend indicates the ratio of distance between neighboring conductors vs. the conductor radius.

Fig. 3 also shows the computation results from the above example. It can be seen that when the D/R ratio is less than 20%, the electric field vanishes from 120 degrees to 240 degrees. It can be seen that this region is located in the area enclosed by four conductors, where the potential must be uniform when *D* is very small. Therefore the electric field is close to zero in this area.

For comparison purposes, the Successive Image Method [2] is also used to study this case. The computation results are shown in Fig. 4.

In Fig. 4, when the D/R ratio is less than 20%, the curve oscillates from 100 degrees to 260 degrees. This is obviously incorrect because the electric field must be zero in an equipotential area.

Comparing the curves for ratio D/R =100% in Fig. 3 and Fig. 4, we can see good agreement. Other curves with D/R ratio larger than 100% are also consistent. This means that when the gaps between the neighboring conductors are larger than the conductor radius, both the Strip Simulation Method and successive image method give similar results.

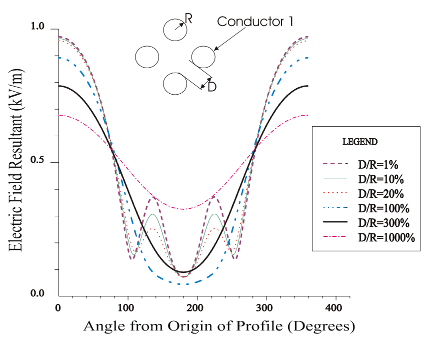

Fig. 4. Electric field on the surface of Conductor 1, computed using the Successive Image Method. The legend indicates the ratio of distance between neighboring conductors vs. the conductor radius.

Looking at the maximum electric fields computed in Figs. 3 and 4 (near 0 degrees), one can see that the maximum electric field evaluated in Fig. 4 is around 4% higher than the one in Fig. 3 for small values of D/R. This is mainly due to the fact that the total charge evaluated in the case of Fig. 3 (Strip Simulation Method) is made with the Markt and Mengele charge evaluation method. This method has a tendency to underestimate the total charge when D/R is small. The error in the evaluation of the charges for D/R = 1% is about 1.6% for the four conductor bundles. The electric field computed at ground level will have the same difference.

In order to compare the computation results of electric fields at other locations when the ratio D/R is greater than 1, a profile is placed on the earth surface, immediately below and perpendicular to the conductors, extending 50 meters symmetrically on both sides of the center line of the conductors. The computation results from both methods are shown in Fig. 5 and it can be seen that they are in good agreement.

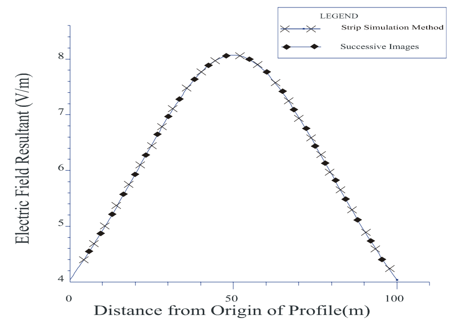

Fig. 5. Electric field on ground surface computed using The Strip Simulation Method and successive image method respectively.

IV. A Practical Example

The Hydro-Quebec second generation 735 kV line consisting of a three-phase transmission line with two shield wires, is shown in Fig. 6 [3]. Each phase is a bundle of four conductors and the shield wire is a single conductor. The phase bundles are 27.43 m high and the shield wire is 40.23 m high. The conductors in the phase bundles are located at the corners of 0.457 m by 0.457 m square. The radius of each conductor is 1.53 cm and the radius of the ground wire is 0.61 cm. The soil resistivity is 100 ohm-meters.

The phases are at735 kV, phase-to-phase, and the phase angles are 0, -120 and 120 degrees, respectively. Each conductor is simulated by a 512-sided polygon for maximum accuracy. Observation Profile 1 and Profile 2 are placed at the surface of the left shield wire and of the upper rightmost conductor of leftmost phase bundle, respectively. Observation Profile 3 is 100 meters long and is placed 1 meter above the earth surface, symmetric with respect to the transmission line center line.

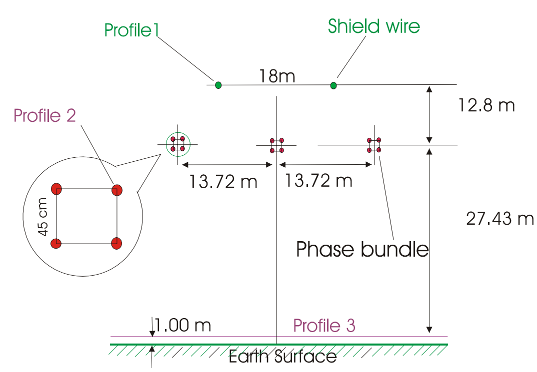

Fig. 6. The cross section of a 3-phase power line with two shield wires and 4-conductor phase bundles.

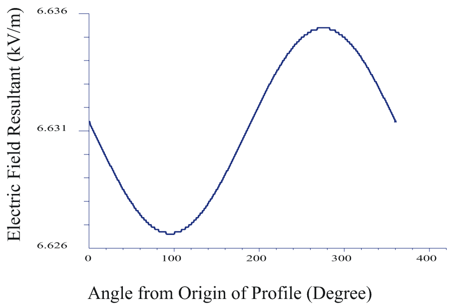

Fig. 7. Electric field on the surface of the left shield wire.

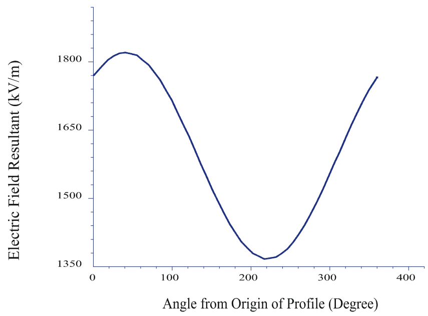

Fig. 8. Electric field on Profile 2.

Fig. 7 shows that the maximum field on the left shield wire occurs at around 270 degrees, beneath the shield wire. Fig. 8 shows the maximum gradient on Profile 2. The maximum field occurs at around 40 degrees, which is at the outside of the bundle. The minimum field occurs at around 220 degrees, which is near the closest point to the center of the bundle.

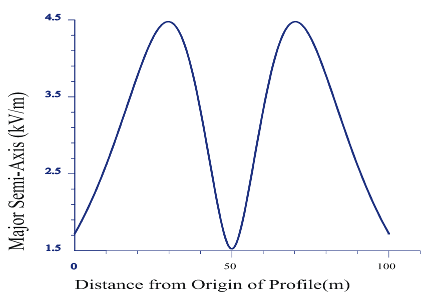

Fig. 9 Major semi-axis of the ellipse of the electric field on Profile 3.

The electric field on Profile 3 is elliptically polarized. Fig. 9 shows the major semi-axis of the ellipse (i.e., the peak value) of the field along Profile 3.

V. Conclusion

Field computations using the Strip Simulation Method and the successive image method have been carried out for bundled transmission line configurations. Comparison of the results obtained for bundles of four conductors shows that when the ratio of the distance between conductors to the conductor radius is smaller than 1, the successive image method tends to overestimate the minimum gradient on the conductor for bundles having more than 2 conductors. The Strip Simulation Method correctly evaluates the distribution of the fields. When the ratio of the distance between conductors to the conductor radius is larger than 1, both methods give accurate results.

VI. Acknowledgments

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VII. References

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VIII. Biographies

Dr. Yixin Yang received the B.Sc., M.Eng. and Ph.D. degrees in 1982, 1985 and 1992 respectively. From 1989 to 1997, he was a senior electronic engineer. From 1997 to 1998, he was a visiting fellow at Griffith University, Australia. Since September 1998, he has been with the R & D Dept. of SES in Montreal. His research interests are in transient electromagnetic scattering, EMI and EMC, and the analysis of grounding systems in various soil structures.

Mr. Daniel Dallaire received the B.Sc.A. degree in electrical engineering in 1971 and the M.Sc.A. degree from École Polytechnique de Montréal in 1981. From 1971 to 1997, he was with the Hydro-Québec "Institut de Recherche" center as a research engineer. From 1998 to 2002, he worked as a consultant engineer in the domain of high frequency electromagnetic interference and grounding. Since 2002, he joined the R & D Department of Safe Engineering Services & technologies in Montreal as a Senior Research Scientist.

His research interests are in transient interfering noise and electromagnetic immunity, EMI and EMC, high frequency grounding and protection, and the conception and the environnemental impact of high voltage lines and stations (Ac or/and Dc.)

Dr. Jinxi Ma

Refer to paper: "Correlation Between Worst Case Safety Conditions And Soil Resistivity Under Power System Fault Conditions"

Dr. Farid Paul Dawalibi

Refer to paper: "Feasibility of Electrical Separation of Proximate Grounding Systems as a Function of Soil Structure"